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Corrigendum

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Sheng-Mei Ao 2005 *J. Phys. A: Math. Gen.* **38** 2399–2413

I would like to express my thanks to Chen *et al* for drawing my attention to their work (*J. Phys. A: Math. Gen.* **31** 6929). In my own paper I attempted to correct Chen *et al*'s work. I would like to apologize to them as in my paper I reproduced the background knowledge from sections 2 and 3 of their paper. I understand that this is unacceptable and I apologize to Chen and his co-workers. My original intention was for the readers to be able to compare the two papers easily.

In this corrigendum I would like to clarify the main differences between the two papers. Chen *et al* used only one squared Jost function $\Phi(z,\zeta)$ in their basis of perturbation expansion, while we used two squared Jost functions $\Phi(z,\zeta)$ and $\Psi(z,\zeta)$ to construct a complete basis. This results in different discrete spectra eigenfunctions, different expansion coefficients and so on. In particular our physical results (for example the soliton velocity and the first-order correction) are quite different from those in Chen *et al*'s paper.

A list of key differences between my paper and Chen *et al*'s paper are given along with some recent corrections (the results with the superscript symbol $(-)$ correspond to those of Chen's paper):

Different bases:

$$B^{(-)} = \{\Phi(z,\zeta), \Phi_0(z), \Phi_1^{(-)}(z)\}, \qquad B = \{\Phi(z,\zeta), \Psi(z,\zeta), \Phi_0(z), \Phi_1(z)\}$$

Different discrete spectra eigenfunctions:

$$\Phi_1^{(-)}(z) = \frac{\mathrm{i}}{2}\begin{pmatrix} \{-\lambda_1(\theta_1 - \frac{1}{2})\mathrm{sech}^2\theta_1 - \zeta_1 \tanh\theta_1 - \zeta_1\}e^{-\mathrm{i}\beta_1} \\ \{\lambda_1(\theta_1 - \frac{1}{2})\mathrm{sech}^2\theta_1 + \overline{\zeta}_1 \tanh\theta_1 + \overline{\zeta}_1\}e^{\mathrm{i}\beta_1} \end{pmatrix},$$

$$\Phi_1(z) = \frac{1}{2}\begin{pmatrix} \{-\mathrm{i}\lambda_1\theta_1\mathrm{sech}^2\theta_1 - \mathrm{i}\lambda_1 \tanh\theta_1 + k_1\}e^{-\mathrm{i}\beta_1} \\ \{\mathrm{i}\lambda_1\theta_1\mathrm{sech}^2\theta_1 + \mathrm{i}\lambda_1 \tanh\theta_1 + k_1\}e^{\mathrm{i}\beta_1} \end{pmatrix}.$$

Different first-order corrections:

$$\mathbf{q}^{(-)}(z,t_0) = \int_{-\infty}^{\infty} G^{(-)}(z,z';t_0)\mathbf{R}(z')\mathrm{d}z', \qquad \mathbf{q}(z,t_0) = \int_{-\infty}^{\infty} G(z,z';t_0)\mathbf{R}(z')\mathrm{d}z',$$

in which $G^{(-)}$ and G are Green's functions defined as

$$G^{(-)}(z,z';t_0) = \int_C \frac{(\mathrm{e}^{\mathrm{i}4\kappa(\lambda-\lambda_1)t_0} - 1)\Phi(z,\zeta)\Phi(z',\zeta)^A}{8\pi\kappa(\lambda-\lambda_1)a(\zeta)^2(1-\rho^2\zeta^{-2})}\mathrm{d}\zeta.$$

$$G(z,z';t_0) = \int_C \frac{(\mathrm{e}^{-\mathrm{i}4\kappa(\lambda-\lambda_1)t_0} - 1)\Psi(z,\zeta)\Psi(z',\zeta)^A}{16\pi\kappa(\lambda-\lambda_1)a(\zeta)^2(1-\rho^2\zeta^{-2})}\mathrm{d}\zeta,$$

$$+ \int_C \frac{(\mathrm{e}^{\mathrm{i}4\kappa(\lambda-\lambda_1)t_0} - 1)\Phi(z,\zeta)\Phi(z',\zeta)^A}{16\pi\kappa(\lambda-\lambda_1)a(\zeta)^2(1-\rho^2\zeta^{-2})}\mathrm{d}\zeta.$$

Different evolution equations of soliton parameters:

$$
\begin{aligned}
(2\mathcal{L}-1)\rho\rho_{t_1} + \lambda_1 k_1 \beta_{1t_1} + 2k_1^3 \xi_{t_1}^{(-)}
&= \int_{-\infty}^{\infty} k_1(1-\tanh\theta_1)\mathrm{Re}(r[u]\mathrm{e}^{\mathrm{i}\beta_1})\mathrm{d}\theta_1 \\
&\quad - \int_{-\infty}^{\infty} \lambda_1\left(\theta_1 \mathrm{sech}^2\theta_1 + \frac{1}{2}\mathrm{sech}^2\theta_1 + \tanh\theta_1 - 1\right)\mathrm{Im}(r[u]\mathrm{e}^{\mathrm{i}\beta_1})\mathrm{d}\theta_1 \ ,
\end{aligned}
$$

$$
\lambda_1 k_1 \beta_{1t_1} + 2k_1^3 \xi_{t_1} = \int_{-\infty}^{\infty} k_1 \mathrm{Re}(r[u]\mathrm{e}^{\mathrm{i}\beta_1})\mathrm{d}\theta_1 - \int_{-\infty}^{\infty} \lambda_1(\theta_1 \mathrm{sech}^2\theta_1 + \tanh\theta_1)\mathrm{Im}(r[u]\mathrm{e}^{\mathrm{i}\beta_1})\mathrm{d}\theta_1
$$

which determine the different corrections of soliton velocity ($\xi_{t_1}^{(-)}$ or ξ_{t_1}).

I would also like to say that I was unaware of the second paper in which the completeness of Chen *et al*'s expansion basis was proven (*J. Phys. A: Math. Gen.* **32** 2399) by a different method from ours by Huang *et al* until after our paper had been published. After being made aware of it I thoroughly checked our paper once again. Apart from some small calculated errors, our discussion was correct and our modified results still differ from Huang *et al*'s paper. I began to wonder why there was more than one physical result for the same problem and therefore carried out a further study to investigate this. I have proved mathematically that the different physical results obtained from starting from different bases are essentially equivalent.

I would like to acknowledge Huang *et al* for their second paper. It has encouraged me to carry out further research to understand the problem.

Finally, I would like to apologize for any offence I may have caused to Professor N N Huang, Dr X-J Chen and Dr Z-D Chen.